

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Sr. Secondary School Supplementary Examination, July- 2023
MATHEMATICS PAPER CODE 65(B)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC.”
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.
4	The Marking scheme carries only suggested value points for the answers. These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note “Extra Question”.</u>
10	<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question”.</u>

QUESTION PAPER CODE 65C(B)

EXPECTED ANSWER/VALUE POINTS

SECTION A

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
SECTION-A (Question nos. 1 to 18 are Multiple choice Questions carrying 1 mark each)		
1.	The domain of the function $\sin^{-1}(2x)$ is : (a) $[-1, 1]$ (b) $[0, 1]$ (c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$	
Ans	(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$	1
2.	If the matrix $\begin{bmatrix} 0 & 3 & 5 \\ k+1 & 0 & 4 \\ -5 & k & 0 \end{bmatrix}$ is a skew symmetric matrix, then the value of k is : (a) 4 (b) 2 (c) -2 (d) -4	
Ans	(d) -4	1
3.	If $A = \begin{bmatrix} 4 & -3 \\ 9 & -3 \end{bmatrix}$, then $ A $ is equal to : (a) 15 (b) -42 (c) 0 (d) 25	
Ans	(a) 15	1
4.	If $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$ and $A + A' = I$, then the value of α is : (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$	
Ans	(b) $\frac{\pi}{3}$	1

5.	If A is a 2×3 matrix and B is another matrix such that both $A'B$ and BA' are defined, then order of B is : (a) 3×2 (b) 2×3 (c) 3×3 (d) 2×2	
Ans	(b) 2×3	1
6.	If $y = 3 \log \sqrt{\sin x}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is : (a) 3 (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$	
Ans	(c) $\frac{3}{2}$	1
7.	An angle θ, $0 < \theta < \frac{\pi}{2}$, which increases twice as fast as its sine, is : (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$	
Ans	(a) $\frac{\pi}{3}$	1
8.	The value of k for which the function f, given by $f(x) = \begin{cases} kx + 1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$ is continuous at $x = \pi$, is : (a) $\frac{2}{\pi}$ (b) $-\frac{\pi}{2}$ (c) $-\frac{2}{\pi}$ (d) $\frac{\pi}{2}$	
Ans	(c) $-\frac{2}{\pi}$	1
9.	If $y = \sin^{-1}(2x\sqrt{1-x^2})$, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$, then $\frac{dy}{dx}$ is : (a) 2 (b) $\frac{2}{\sqrt{1-x^2}}$ (c) $\frac{-2}{\sqrt{1-x^2}}$ (d) $\sqrt{1-x^2}$	

Ans	(b) $\frac{2}{\sqrt{1-x^2}}$	1
10.	The rate of change of the volume of a spherical bubble with respect to its radius r at $r = 3$ cm is : (a) $24 \pi \text{ cm}^3/\text{cm}$ (b) $36 \pi \text{ cm}^2/\text{cm}$ (c) $36 \pi \text{ cm}^3/\text{cm}$ (d) $24 \pi \text{ cm}^2/\text{cm}$	
Ans	(c) $36\pi \text{ cm}^3 / \text{cm}$	1
11.	$\int \cot^2 x \, dx$ equals : (a) $-\cot x + x + C$ (b) $-\cot x - x + C$ (c) $\cot x - x + C$ (d) $\cot x + x + C$	
Ans	(b) $-\cot x - x + C$	1
12.	The value of $\int_{-1}^1 \frac{dx}{5 + 2x + x^2}$ is : (a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$	
Ans	(a) $\frac{\pi}{8}$	1
13.	The sum of the order and the degree of the differential equation $3x^2 \left(\frac{d^2y}{dx^2} \right)^3 + 5 \frac{d^3y}{dx^3} + 9 = 0$ is : (a) 5 (b) 6 (c) 4 (d) 2	
Ans	(c) 4	1
14.	A particular solution of the differential equation $dy = y \tan x \, dx$; $y = 1$ when $x = 0$, is : (a) $y = 0$ (b) $y = 1 + \sec x$ (c) $y = \sec x$ (d) $y \cos x = 0$	
Ans	(c) $y = \sec x$	1

15.	If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then the value of θ is : (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$	
Ans	(a) $\frac{\pi}{3}$	1
16.	The position vectors of two points A and B are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ respectively. The position vector of a point C which divides AB externally in the ratio 1 : 2 is : (a) $-3\vec{a} - 5\vec{b}$ (b) $-7\vec{b}$ (c) $\frac{1}{3}(5\vec{a} - \vec{b})$ (d) $(3\vec{a} + 5\vec{b})$	
Ans	(d) $(3\vec{a} + 5\vec{b})$	1
17.	The value of λ for which the points A, B and C having position vectors $(3\hat{i} - 2\hat{j} + 4\hat{k})$, $(\hat{i} + \lambda\hat{j} + \hat{k})$ and $(-\hat{i} + 4\hat{j} - 2\hat{k})$ respectively are collinear, is : (a) 4 (b) 1 (c) 3 (d) 2	
Ans	(b) 1	1
18.	The direction ratios of a line whose Cartesian equations are $3x - 3 = 2y + 1 = 5 - 6z$, are : (a) 2, 3, -1 (b) 3, -2, 1 (c) 2, 1, -3 (d) 3, 2, -1	
Ans	(a) 2,3,-1	1
(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)		
19.	<i>Assertion (A) :</i> The vector equation of a line passing through the points (1, 2, 3) and (3, 0, 2) is $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} - \hat{k})$. <i>Reason (R) :</i> Equation of a line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.	

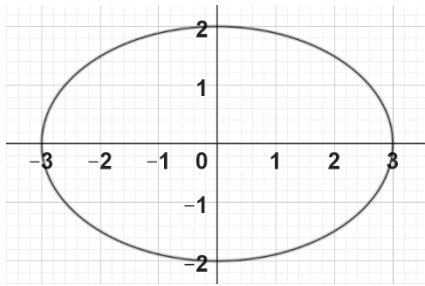
Ans	(a) Both Assertion (A) and Reason (R) are true, and Reason(R) is the correct explanation of the Assertion (A).	1
20.	<i>Assertion (A) :</i> Given two independent events A and B such that $P(A) = 0.3$ and $P(B) = 0.6$, then $P(A \text{ and not } B) = 0.12$. <i>Reason (R) :</i> For two independent events A and B, $P(A \text{ and } B) = P(A) \cdot P(B)$.	
Ans	(a) Both Assertion (A) and Reason (R) are true, and Reason(R) is the correct explanation of the Assertion (A).	1
SECTION-B (Question nos. 21 to 25 are very short Answer type questions carrying 2 marks each)		
21.	(a) Let the relation R be given as $R = \{(x, y) : x, y \in \mathbb{N} \text{ and } x + 3y = 12\}$. Find the domain and range of R. OR (b) Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^4$ is neither one-one nor onto.	
Ans	(a) $y = \frac{12-x}{3}; x, y \in \mathbb{N}$ Domain = $\{3, 6, 9\}$ Range = $\{1, 2, 3\}$ Or (b) $f(-1) = f(1) = 1$ but $-1 \neq 1$, $\therefore$ 'f' is not a one-one function. $0 \leq x^4 < \infty \Rightarrow R_f = [0, \infty) \Rightarrow \text{Range} \neq \text{Co-domain},$ $\therefore$ 'f' is not an onto function.	1 1 1 1
22.	Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.	
Ans	Symmetric = $\frac{1}{2}(A + A') = \begin{bmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix}$ Skew-Symmetric = $\frac{1}{2}(A - A') = \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$	1 $\frac{1}{2}$ $\frac{1}{2}$

23.	If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.	
Ans	$y = x^x \Rightarrow \log y = x \log x$, differentiating with respect to 'x', we get $\frac{dy}{dx} = y(1 + \log x)$, differentiating with respect to 'x', we get $\frac{d^2y}{dx^2} = \frac{y}{x} + (1 + \log x) \frac{dy}{dx} \Rightarrow \frac{d^2y}{dx^2} = \frac{y}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right)^2 \Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$	1 1
24.	Express the vector $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b} = (3\hat{i} + \hat{k})$ and the other is perpendicular to $\vec{b}$.	
Ans	Let $\vec{a} = \vec{c} + \vec{d}$, $\vec{c} \parallel \vec{b} \Rightarrow \vec{c} = \lambda \vec{b} \therefore \vec{c} = 3\lambda\hat{i} + \lambda\hat{k}$ and $\vec{d} = (5 - 3\lambda)\hat{i} - 2\hat{j} + (5 - \lambda)\hat{k}$ $\vec{b} \cdot \vec{d} = 0 \Rightarrow 15 - 9\lambda + 5 - \lambda = 0 \Rightarrow \lambda = 2$ $\vec{a} = (6\hat{i} + 2\hat{k}) + \lambda(-\hat{i} - 2\hat{j} + 3\hat{k})$	1 $\frac{1}{2}$ $\frac{1}{2}$
25.	(a) Find the angle between the pair of lines given by $\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z+4}{2}$; $\frac{x-5}{-3} = \frac{y+2}{2} = \frac{z}{6}$. OR (b) If the lines $\frac{x-1}{-3} = \frac{2y-2}{4k} = \frac{3-z}{-2}$ and $\frac{x-1}{3k} = \frac{3y-1}{6} = \frac{z-6}{-5}$ are perpendicular to each other, find the value of k.	
Ans	(a) D-ratios of the two lines are 1, -2, 2 and -3, 2, 6 $\cos \theta = \frac{-3-4+12}{3 \cdot 7} = \frac{5}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{5}{21}\right)$ Or (b) The equation of the lines in standard form are. $\frac{x-1}{-3} = \frac{y-1}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-\frac{1}{3}}{2} = \frac{z-6}{-5}$ Lines are perpendicular $\therefore -9k + 4k - 10 = 0 \Rightarrow k = -2$	1 1 1 1

SECTION-C (Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)		
26.	Find the intervals in which the function given by $f(x) = \sin 3x$, $x \in \left[0, \frac{\pi}{2}\right]$ is (a) increasing (b) decreasing.	
Ans	(a) $f(x) = \sin 3x \Rightarrow f'(x) = 0 \Rightarrow 3 \cos 3x = 0 \Rightarrow x = \frac{\pi}{6}, \frac{\pi}{2}$ $f'(x) \geq 0, \forall x \in \left[0, \frac{\pi}{6}\right] \Rightarrow f(x)$ is increasing on $\left[0, \frac{\pi}{6}\right]$ $f'(x) \leq 0, \forall x \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right] \Rightarrow f(x)$ is decreasing on $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$	1 1 1
27.	Find : $\int \frac{x-5}{(x-2)^4} e^x dx$	
Ans	$\int e^x \frac{x-5}{(x-2)^4} dx = \int e^x \left[\frac{(x-2)-3}{(x-2)^4} \right] dx$ $= \int e^x \left[\frac{1}{(x-2)^3} - \frac{3}{(x-2)^4} \right] dx$ $= \frac{e^x}{(x-2)^3} + C$	1 1 1
28.	(a) Find : $\int \frac{2}{(1-x)(1+x^2)} dx$ OR (b) Find : $\int \sqrt{3-2x-x^2} dx$	
Ans	(a) $\int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x+1}{1+x^2} dx$ $= -\log 1-x + \frac{1}{2} \log(1+x^2) + \tan^{-1} x + C$ Or (b) $\int \sqrt{3-2x-x^2} dx = \int \sqrt{2^2 - (x+1)^2} dx$	$1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$

	$= \frac{(x+1)\sqrt{3-2x-x^2}}{2} + 2\sin^{-1}\frac{x+1}{2} + C$	
29.	Evaluate : $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$	
Ans	Let $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx = \int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx$, $I = \int_0^{\pi} \frac{(\pi-x) \sin x}{1 + \sin x} dx$ $\Rightarrow 2I = \pi \int_0^{\pi} \frac{\sin x}{1 + \sin x} dx = \pi \int_0^{\pi} \left[1 - \frac{1}{1 + \sin x} \right] dx = \pi \int_0^{\pi} \left[1 - \frac{1}{1 + \cos\left(\frac{\pi}{2} - x\right)} \right] dx$ $\Rightarrow 2I = \pi \int_0^{\pi} \left[1 - \frac{1}{2 \cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)} \right] dx = \pi \int_0^{\pi} \left[1 - \frac{1}{2} \sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right) \right] dx$ $\Rightarrow 2I = \pi \left[x + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) \right]_0^{\pi} = \pi(\pi - 2), \therefore I = \frac{\pi}{2}(\pi - 2)$	1 $\frac{1}{2}$ $\frac{1}{2}$ 1
30.	(a) Solve the differential equation $x \frac{dy}{dx} = y - x \tan \frac{y}{x}$. OR (b) Find the particular solution of the differential equation $x \frac{dy}{dx} + y = x \cos x + \sin x$; given $y\left(\frac{\pi}{2}\right) = 1$.	
Ans	(a) The given differential equation can be written as, $\frac{dy}{dx} = \frac{y}{x} - \tan \frac{y}{x}$ Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$ $\therefore v + x \frac{dv}{dx} = v - \tan v$ $\Rightarrow \int \frac{dv}{\tan v} = - \int \frac{dx}{x}$ $\Rightarrow \log \sin v = -\log x + \log C$ $\Rightarrow \sin \frac{y}{x} = \frac{C}{x} \Rightarrow x \sin \frac{y}{x} = C$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$

	Or (b) The given differential equation can be written as $\frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{\sin x}{x}$ The integrating factor, I.F. = $e^{\int \frac{1}{x} dx} = e^{\log x} = x$ $\therefore$ the solution is: $y \cdot x = \int (x \cos x + \sin x) dx$ $= x \sin x - \int \sin x dx + \int \sin x dx$ $y \cdot x = x \sin x + C \Rightarrow y = \sin x + \frac{C}{x}$ Put, $x = \frac{\pi}{2}, y = 1 \Rightarrow C = 0$ $\therefore$ the particular solution is $y = \sin x$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$												
31.	(a) Let A and B be the events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not A or not B}) = \frac{1}{4}$. Find whether A and B are (i) mutually exclusive (ii) independent. OR (b) Find the mean of the number of tails in two tosses of a coin.													
Ans	(a) $P(A' \cup B') = \frac{1}{4}$ $\Rightarrow P(A \cap B)' = \frac{1}{4}$ $\Rightarrow 1 - P(A \cap B) = \frac{1}{4} \Rightarrow P(A \cap B) = \frac{3}{4} \neq 0, \therefore \text{A and B are not mutually exclusive.}$ $P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{7}{12} = \frac{7}{24} \neq P(A \cap B), \therefore \text{A and B are not independent.}$ Or (b) Let X = Number of tails, then <table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{1}{4}$</td><td>$\frac{2}{4}$</td><td>$\frac{1}{4}$</td></tr><tr><td>X · P(X)</td><td>0</td><td>$\frac{2}{4}$</td><td>$\frac{2}{4}$</td></tr></table> Mean number of tails = $\mu = \Sigma X \cdot P(X) = 0 + \frac{2}{4} + \frac{2}{4} = 1$	X	0	1	2	P(X)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	X · P(X)	0	$\frac{2}{4}$	$\frac{2}{4}$	1 1 1 $\frac{1}{2}$ $1 \frac{1}{2}$ 1
X	0	1	2											
P(X)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$											
X · P(X)	0	$\frac{2}{4}$	$\frac{2}{4}$											

SECTION-D (Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)		
32.	(a) Let R be a relation in $\mathbb{R}$, the set of all real numbers, defined by $R = \{(a, b) : a \leq b^3\}$. Show that R is neither reflexive, nor symmetric and nor transitive. OR (b) Let set $A = \{1, 2, 3, \dots, 10\}$ and R be a relation in $A \times A$, defined by $(a, b) R (c, d) \Leftrightarrow a + d = b + c$ for all (a, b) and $(c, d) \in A \times A$. Prove that R is an equivalence relation.	
Ans	(a) $\frac{1}{2} \not\leq \left(\frac{1}{2}\right)^3 \therefore R$ is not reflexive $1 \leq 2^3 \Rightarrow (1, 2) \in R$ but $2 \not\leq 1^3 \Rightarrow (2, 1) \notin R, \therefore R$ is not symmetric. $9 \leq 3^3, 3 \leq 2^3$ but $9 \not\leq 2^3 \Rightarrow (9, 3), (3, 2) \in R$ but $(9, 2) \notin R, \therefore R$ is not transitive. Or (c) $a + b = b + a \Rightarrow (a, b) R (a, b), \forall (a, b) \in R, \therefore R$ is reflexive $(a, b) R (c, d) \Rightarrow a + d = b + c \Rightarrow c + b = d + a \Rightarrow (c, d) R (a, b), \therefore R$ is symmetric $(a, b) R (c, d) \& (c, d) R (e, f) \Rightarrow a + d = b + c ; c + f = d + e,$ Adding the two equations $\Rightarrow a + f = b + e, \therefore (a, b) R (e, f)$ $\Rightarrow R$ is a transitive relation $\therefore R$ is an equivalence relation	$1\frac{1}{2}$ $1\frac{1}{2}$ 2 $1\frac{1}{2}$ $1\frac{1}{2}$ 2
33.	Using integration, find the area enclosed by the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$.	
Ans	 Required area $= 4 \cdot \frac{2}{3} \int_0^3 \sqrt{9 - x^2} dx$ $= \frac{8}{3} \left[\frac{x\sqrt{9-x^2}}{2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_0^3$ $= \frac{8}{3} \left[0 + \frac{9\pi}{4} \right] = 6\pi$	2 $1\frac{1}{2}$ $1\frac{1}{2}$

34.	While solving the linear programming problem Minimise and Maximise $Z = 3x + 9y$, subject to the constraints $x + 3y \leq 60$, $x + y \geq 10$, $x \leq y$ and $x \geq 0$, $y \geq 0$ graphically, the corner points of the feasible region ABCD are A(0, 10), B(5, 5), C(15, 15) and D(0, 20). Find the minimum value and the maximum value of Z along with the corresponding corner points.	
Ans	$Z(A) = 90$; $Z(B) = 60$; $Z(C) = 180$; $Z(D) = 180$ $\text{Min}(Z) = 60$ at B(5, 5) ; $\text{Max}(Z) = 180$ at C(15, 15) and D(0, 20)	2 $1\frac{1}{2} + 1\frac{1}{2}$
35.	(a) Show that the lines $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = \hat{i} + \hat{j} + \hat{k} + \mu(\hat{i} - \hat{j} + 2\hat{k})$ do not intersect. OR (b) Find the coordinates of the foot of perpendicular drawn from the point (3, -1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ Hence, write the equation of this perpendicular line.	
Ans	(a) $(\vec{a}_2 - \vec{a}_1) = -\hat{j}$ and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = -\hat{i} - \hat{j}$ $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 1 \neq 0 \Rightarrow \text{Lines are not intersecting}$ Or (b) General point on the line, say, P(2λ, 3λ + 2, 4λ + 3) Direction ratios of the perpendicular from the point (3, -1, 11) to the line are 2λ - 3, 3λ + 3, 4λ - 8 And direction ratios of the line are 2, 3, 4 $\therefore 2(2\lambda - 3) + 3(3\lambda + 3) + 4(4\lambda - 8) = 0 \Rightarrow \lambda = 1$, $\therefore (2, 5, 7)$ is the foot of the perpendicular Equation of the perpendicular Cartesian form: $\frac{x-3}{1} = \frac{y+1}{-6} = \frac{z-11}{4}$	1+1 2+1 1 1 1 1

	Or, Vector form: $\vec{r} = (3\hat{i} - \hat{j} + 11\hat{k}) + \lambda(\hat{i} - 6\hat{j} + 4\hat{k})$	
SECTION-E (Question nos. 36 to 38 are source based/case based/passages based/integrated units of assessment questions carrying 4 marks each)		
36.	A professional typist having his shop in a busy market charges ₹ 200 for typing 8 English and 4 Hindi pages, while he charges ₹ 275 for typing 5 English and 10 Hindi pages. Based on the above information, answer the following questions : (i) If he charges ₹ x for one page of English and ₹ y for one page of Hindi, express the above as a pair of linear equations. (ii) Express the information in terms of matrix equation $AX = B$. (iii) (a) Find A. OR (iii) (b) Find $(\text{adj } A)$.	
Ans	(i) $8x + 4y = 200$; $5x + 10y = 275$ (ii) $\begin{bmatrix} 8 & 4 \\ 5 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 275 \end{bmatrix}$ (iii)(a) $A = 60$ Or (iii)(b) $\text{adj}A = \begin{bmatrix} 10 & -4 \\ -5 & 8 \end{bmatrix}$	1 1 2 2

37.	Ravindra started to run a small factory of manufacturing LED bulbs. He can sell x bulbs at a price of ₹ $(300 - x)$ each. The cost price of x bulbs is ₹ $(2x^2 - 60x + 18)$. Based on the above information, answer the following questions : (i) Find the profit function $P(x)$ for selling x bulbs. (ii) What is $\frac{d}{dx} [P(x)]$? (iii) (a) How many bulbs should he sell to earn maximum profit ? OR (iii) (b) How many bulbs is he selling if he is incurring a loss of ₹ 18 ?	
Ans	(i) $P(x) = R(x) - C(x) = x(300 - x) - (2x^2 - 60x + 18) = -3x^2 + 360x - 18$ (ii) $\frac{d}{dx} [P(x)] = -6x + 360$ (iii) (a) $\frac{dP}{dx} = 0 \Rightarrow x = 60; \left. \frac{d^2P}{dx^2} \right _{x=60} = -6 < 0$ $\Rightarrow$ Profit is maximum when he sells 60 bulbs Or (iii) (b) $P(x) = -18 \Rightarrow -3x^2 + 360x - 18 = -18 \Rightarrow x = 120$	1 1 1 1 2

38.	A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%. Based on the above information, answer the following questions : (i) What is the probability of a randomly chosen seed to germinate ? (ii) What is the probability that the randomly selected seed is of type A_1, given that it germinates ?	
Ans	(i) Probability of randomly chosen seed to germinate $= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100} = 0.49$ (ii) Probability that the randomly selected seed is of type A_1, given that it germinates $= \frac{\frac{4}{10} \times \frac{45}{100}}{\frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}} = \frac{18}{49}$	2 2